

# Vaidya Spacetime for Galileon Gravity's Rainbow

Prabir Rudra<sup>1\*</sup>, Mir Faizal<sup>2 †</sup> and Ahmed Farag Ali<sup>3‡</sup>

<sup>1</sup>*Department of Mathematics, Asutosh College, Kolkata-700 026, India.*

<sup>2</sup>*Department of Physics and Astronomy, University of Lethbridge, Lethbridge, Alberta, T1K 3M4, Canada*

<sup>3</sup>*Department of Physics, Faculty of Science, Benha University, Benha, 13518, Egypt.*

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## Abstract

In this paper, we analyze Vaidya spacetime with an energy dependent metric in Galileon gravity's rainbow. This will be done using the rainbow functions which are motivated from the results obtained in loop quantum gravity approach and noncommutative geometry. We will investigate the Gravitational collapse in this Galileon gravity's rainbow. We will discuss the behavior of singularities formed from the gravitational collapse in this rainbow deformed Galileon gravity.

## 1 Introduction

The observations from type I supernovae indicate that our universe has a positive cosmological constant and is accelerating in its expansion [1]- [6]. Furthermore, it is known that general theory of relativity has not tested at very large or very small scales, and it is possible for the general theory of relativity to be modified at such scales. However, as gravity has been thoroughly tested at the scale of solar system, it is important for any theory of modified gravity to reduce to the general theory of relativity at the scale of the solar system. It may be noted an interesting model of modified gravity is called the DGP brane model has been proposed to explain accelerating cosmic expansion [7]. This model has two branches and one these branches admits a self-accelerating solution. However, this model contains ghost instabilities, and thus cannot be used as a physical model for the cosmic acceleration [8]. It may be noted that such instabilities also occur for other models of modified gravity [9]. Such

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\*prudra.math@gmail.com

†mirfaizalmir@gmail.com

‡ahmed.ali@fsc.bu.edu.eg

instabilities occur due to the introduction of extra degrees of freedom into the theory because of the existence of higher derivative terms.

However, it is possible to construct an infrared modification of general theory of relativity [10]. This theory contains a self-interaction term of the form  $(\nabla\phi)^2 \propto \phi$ , and so general relativity is recovered at high densities. It is interesting to note that in the Minkowski background, this theory is invariant under the Galileon shift symmetry,  $\delta_\mu\phi \rightarrow \delta_\mu\phi + c_\mu$ . This symmetry prevents the occurrence of higher derivative terms in the equation of motion of this theory. As this theory does not contain extra degrees of freedom, it cannot also contain ghost instabilities. The coupling between a Galileon scalar field and massive gravity through composite metrics has also been studied [11]. A full set of equations of motion for a flat Friedmann-Robertson-Walker background were obtained in this theory. The cosmology has also been studied using Galileon gravity, and this has been done by analyzing the linear perturbation in Galileon gravity [12]. Furthermore, low density stars with slow rotation and static relativistic stars have also been analyzed using Galileon gravity [13]. It was observed that the scalar field solution ceases to exist above a critical density, and this corresponds to the maximum mass of a neutron star. The spherical collapse has also been analyzed in the Galileon gravity [14]- [15]. This was done by analyzing the solutions to the Einstein equations in Galileon gravity. Then these solution were used for discussing the conditions for the formation of a black hole or a naked singularity in Galileon gravity. In this paper, we shall perform such an analysis in a theory which combines Galileon gravity with gravity's rainbow.

Another interesting modification to general relativity is called the Horava-Lifshitz gravity [16]- [17]. This theory of gravity is obtained from a UV completion of general relativity, such that general relativity is recovered in the IR limit [16]- [17]. This is done by taking different Lifshitz scaling for space and time. Such a different Lifshitz scaling for space and time has also been taken in type IIA string theory [18], type IIB string theory [19], AdS/CFT correspondence [20]- [23], dilaton black branes [24]- [25], and dilaton black holes [26]- [27]. The Horava-Lifshitz gravity is based on the modification of the usual energy-momentum dispersion relation in the UV limit such that it reduces to the usual energy-momentum dispersion relation in the IR limit. The gravity's rainbow is another modification of gravity based on such a modified energy-momentum dispersion relation in the UV limit [28]- [30]. In gravity's rainbow the metric depends on the energy of the test particle used to probe the structure of the spacetime. The gravity's rainbow can be related to the Horava-Lifshitz gravity, for a specific choice of rainbow functions [31]. There is a strong motivation to study such theories based on the energy-momentum dispersion relation in the UV limit. This is because the Lorentz symmetry fixes the form of the energy-momentum relations, and there are strong theoretical indications from various different approaches to quantum gravity that Lorentz symmetry might only be a symmetry of the low energy effective field theory, and so it will break in the UV limit [32]- [36]. This is expected to occur in discrete spacetime [37], models

based on string field theory [38], spacetime foam [39], the spin-network in loop quantum gravity (LQG) [40], and non-commutative geometry [41]. It may be noted that such a deformation of the standard energy-momentum dispersion relation in the UV limit of the theory leads the existence of a maximum energy scale. The doubly special relativity is build on the existence of such a maximum energy scale [42], and gravity's rainbow is the generalization of doubly special relativity to curved spacetime [43]. In gravity's rainbow, the metric describing the geometry of spacetime depend on the energy of the test particle used to probe the structure of that spacetime. So, the geometry of spacetime is represented by a family of energy dependent metrics forming a rainbow of metrics. In gravity's rainbow, the energy-momentum dispersion relation is modified by energy dependent rainbow functions,  $F(E)$  and  $G(E)$ , such that

$$E^2 F^2(E) - p^2 G^2(E) = m^2. \quad (1)$$

As it is required that the usual energy-momentum dispersion relation is recovered in the IR limit, these rainbow functions are required to satisfy

$$\lim_{E/E_P \rightarrow 0} F(E) = 1, \quad \lim_{E/E_P \rightarrow 0} G(E) = 1. \quad (2)$$

The energy dependent metric in gravity's rainbow can be written as

$$g^{\mu\nu}(E) = \eta^{ab} e_a^\mu(E) e_b^\nu(E). \quad (3)$$

The rainbow functions are defined using the energy  $E$ , which is the energy at which the spacetime is probed, and this energy cannot exceed the Planck energy  $E_P$ .

Vaidya spacetime is a a non-stationary Schwarzschild spacetime [44]- [45]. The gravitational collapse in Vaidya spacetime has been studied in Galileon gravity [14]. In this paper, we will analyze the gravitational collapse in Vaidya spacetime in Galileon gravity deformed by rainbow functions. The gravitational collapse has also been studied in gravity's rainbow [46]- [47]. In fact, the thermodynamics of black holes has also been discussed in gravity's rainbow [48]- [50]. This has been done by deforming the black hole metric by rainbow functions. The energy  $E$  used to define the rainbow functions can be identified with the energy of quantum particle in the vicinity of the event horizon, which could be emitted in the Hawking radiation. It is possible to obtain a bound on this energy  $E \geq 1/\Delta x$ , using the uncertainty principle  $\Delta p \geq 1/\Delta x$ . Here the uncertainty in position of a particle in the vicinity of the event horizon can be equated with the radius of the event horizon radius

$$E \geq 1/\Delta x \approx 1/r_+. \quad (4)$$

The existence of this bound on the energy modifies the temperature of the black hole in gravity's rainbow. This modified temperature of the black hole has been used for calculate the corrected entropy of a black hole in gravity's rainbow. This deformation of the black hole

thermodynamics lead to the formation of black remnants, and these black remnants can have important phenomenological implication for the detection of mini black holes at the LHC [51]. It may be noted that this energy which is used in constructing rainbow functions dynamically dependent on the coordinate [31]. Even though we do not need this explicit dependence of this energy on the coordinate, but it is important to note that the rainbow functions are dynamical, and so they cannot be gauged away.

## 2 Field Equations and the Solutions in Vaidya Space-time in the background of Galileon Gravity

The Galileon theory is invariant under the Galileon shift symmetry. Now if  $\mathcal{L}_m$  is the matter Lagrangian and  $\phi$  is the Galileon field, then the action for such a theory can be written as [10]- [55],

$$S = \int d^4x \sqrt{-g} \left[ \phi R - \frac{w}{\phi} (\nabla\phi)^2 + f(\phi) \square \phi (\nabla\phi)^2 + \mathcal{L}_m \right], \quad (5)$$

where  $w$  is the Galileon parameter, and the coupling  $f(\phi)$  has dimension of length. Furthermore, we also have  $(\nabla\phi)^2 = g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$ , and  $\square \phi = g^{\mu\nu} \nabla_\mu \nabla_\nu \phi$ . Now for a spherically symmetric spacetime, we can write [44],

$$ds^2 = - \left( 1 - \frac{m(t, r)}{r} \right) dt^2 + 2dt dr + r^2 d\Omega_2^2. \quad (6)$$

Here the radial coordinate is denoted by  $r$  and the null coordinate is denoted by  $t$ . The gravitational mass inside the sphere of radius  $r$  is denoted by  $m(t, r)$ , and the line element on a unit 2-sphere is denoted by  $d\Omega_2^2$ .

The Rainbow deformations the above metric can be written as

$$ds^2 = - \frac{1}{F^2(E)} \left( 1 - \frac{m(t, r)}{r} \right) dt^2 + \frac{1}{F(E)G(E)} dt dr + \frac{1}{G^2(E)} r^2 d\Omega_2^2. \quad (7)$$

The Einstein's equations for this metric can be written as

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{2\phi} + \frac{1}{\phi} (\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \square \phi) + \frac{\omega}{\phi^2} \left[ \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla\phi)^2 \right] - \frac{1}{\phi} \left\{ \frac{1}{2} g_{\mu\nu} \nabla_\lambda [f(\phi) (\nabla\phi)^2] \nabla^\lambda \phi - \nabla_\mu [f(\phi) (\nabla\phi)^2] \nabla_\nu \phi + f(\phi) \nabla_\mu \phi \nabla_\nu \phi \square \phi \right\} \quad (8)$$

where  $T_{\mu\nu}$  is the energy momentum tensor.

The energy-momentum tensor for the Vaidya null radiation is given by

$$T_{\mu\nu}^{(n)} = \sigma l_\mu l_\nu, \quad (9)$$

where  $\sigma$  is the energy density corresponding to Vaidya null radiation. The energy-momentum tensor for a perfect fluid is given by

$$T_{\mu\nu}^{(m)} = (\rho + p)(l_\mu \eta_\nu + l_\nu \eta_\mu) + p g_{\mu\nu}, \quad (10)$$

where  $\rho$  and  $p$  are the energy density and pressure for the perfect fluid. Now we can write [56]

$$T_{\mu\nu} = T_{\mu\nu}^{(n)} + T_{\mu\nu}^{(m)} \quad (11)$$

It may be noted that  $l_\mu$  and  $\eta_\mu$  are linearly independent future pointing null vectors,

$$l_\mu = (1, 0, 0, 0) \quad \text{and} \quad \eta_\mu = \left( \frac{1}{2} \left( 1 - \frac{m}{r} \right), -1, 0, 0 \right). \quad (12)$$

Furthermore, they satisfy

$$l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0, \quad l_\lambda \eta^\lambda = -1 \quad (13)$$

Now we can write the Einstein field equations ( $G_{\mu\nu} = T_{\mu\nu}$ ) for the metric (7), and the wave equation for the Galileon field  $\phi$ . Thus, we can use  $G_{00} = T_{00}$ , to obtain

$$\begin{aligned} & \frac{G(E) [G(E)m \{3 - 4m'\} + r \{-3G(E) + 4G(E)m' + 2F(E)\dot{m}\}]}{F(E)2r^3} = \frac{\sigma + \rho \left(1 - \frac{m}{r}\right)}{2\phi} \\ & + \frac{1}{\phi} \left[ \ddot{\phi} - \left( \frac{m}{2r^2} - \frac{m'}{2r} \right) \dot{\phi} - \left( \frac{m}{2r^2} - \frac{m^2}{2r^3} - \frac{m'}{2r} + \frac{mm'}{2r^2} + \frac{\dot{m}}{2r} \right) \phi' \right. \\ & + \left( 1 - \frac{m}{r} \right) \left\{ 2\dot{\phi}' - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \left( 1 - \frac{m}{r} \right) \phi'' \right\} \Big] \\ & + \frac{\omega}{\phi^2} \left[ \dot{\phi}^2 + \frac{1}{2} \left( 1 - \frac{m}{r} \right) \phi' \left( 2\dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi' \right) \right] \\ & + \frac{1}{\phi} \left[ \frac{1}{2} \left( 1 - \frac{m}{r} \right) \left\{ \phi' \nabla_0 U + \left( \dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi' \right) \nabla_1 U \right\} \right. \\ & \left. - \dot{\phi} \nabla_0 U + f(\phi) \dot{\phi}^2 \left\{ 2\dot{\phi}' - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) \left( 1 - \frac{m}{r} \right) \phi'' \right\} \right] \end{aligned} \quad (14)$$

We can use  $G_{11} = T_{11}$ , to obtain

$$\begin{aligned} & \frac{\phi''}{\phi} + \frac{\omega \phi'^2}{\phi^2} - \frac{1}{\phi} \left[ -f(\phi) \left\{ 2\phi'^2 \dot{\phi}' + 2\dot{\phi} \phi' \phi'' + 2 \left( 1 - \frac{m}{r} \right) \phi'^2 \phi'' + \frac{m}{r^2} \phi'^3 \right\} \right. \\ & - f'(\phi) \left\{ 2\phi'^3 \dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi'^4 \right\} \\ & \left. + f(\phi) \phi'^2 \left\{ 2\dot{\phi}' - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \left( 1 - \frac{m}{r} \right) \phi'' \right\} \right] = 0, \end{aligned} \quad (15)$$

We can use  $G_{01} = T_{01}$ , to obtain

$$\begin{aligned}
& \frac{G(E) \{4m' - 3\}}{2r^2 F(E)} \\
&= \frac{\rho}{2\phi} + \frac{1}{\phi} \left[ \dot{\phi}' + \phi' \left( \frac{m'}{2r} - \frac{m}{2r^2} \right) - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \phi'' \left( 1 - \frac{m}{r} \right) \right] \\
&+ \frac{\omega}{2\phi^2} \left( 1 - \frac{m}{r} \right) \phi'^2 + \frac{1}{\phi} \left[ \frac{1}{2} \nabla_0 U \left( \dot{\phi} - 2\phi' \right) + \frac{1}{2} \phi' \nabla_1 U \right. \\
&\left. + f(\phi) \dot{\phi} \phi' \left\{ 2\dot{\phi}' - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \left( 1 - \frac{m}{r} \right) \phi'' \right\} \right], \tag{16}
\end{aligned}$$

We can use  $G_{22} = T_{22}$ , to obtain

$$\begin{aligned}
2rm'' &= \frac{\omega}{\phi^2} \left[ \frac{r^2}{2} \phi' \left\{ 2\dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi' \right\} \right] \\
&- \frac{1}{\phi} \left[ r^2 \left\{ \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) - \left( 1 - \frac{m}{r} \right) \phi'' - 2\dot{\phi}' \right\} \right] \\
&+ \frac{r^2}{2\phi} \left( \nabla_0 U \dot{\phi} + \nabla_1 U \phi' \right) - \frac{pr^2}{2\phi}. \tag{17}
\end{aligned}$$

Finally, we can use  $G_{33} = T_{33}$ , to obtain

$$\begin{aligned}
\frac{pr^2}{2\phi} &+ \frac{1}{\phi} \left[ r\dot{\phi} - (m-r)\phi' - r^2 \left\{ 2\dot{\phi}'\phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \left( 1 - \frac{m}{r} \right) \phi'' \right\} \right] \\
&- \frac{\omega}{\phi^2} \left[ \frac{\phi'}{2} r^2 \left( 2\dot{\phi} + \phi' \left( 1 - \frac{m}{r} \right) \right) \right] - \frac{1}{2\phi} r^2 \left( \dot{\phi} \nabla_0 U + \phi' \nabla_1 U \right) + 2rm'' = 0. \tag{18}
\end{aligned}$$

Here the differentiation with respect to  $t$  is denoted by a over-dot and differentiation with respect  $r$  is denoted by a dash. It is useful to define  $U = f(\phi) (\nabla\phi)^2$ . Now we can write an expression for  $\nabla_0 U$  as

$$\nabla_0 U = f(\phi) \left[ 2\phi' \ddot{\phi} + 2\dot{\phi} \phi' + \left( 1 - \frac{m}{r} \right) 2\phi' \dot{\phi}' - \phi'^2 \frac{\dot{m}}{r} \right] + f'(\phi) \left[ 2\phi' \dot{\phi}^2 + \left( 1 - \frac{m}{r} \right) \phi'^2 \dot{\phi} \right] \tag{19}$$

and an expression for  $\nabla_1 U$  as

$$\nabla_1 U = f(\phi) \left[ 2\phi' \dot{\phi}' + 2\dot{\phi} \phi'' + 2 \left( 1 - \frac{m}{r} \right) \phi' \phi'' + \frac{m}{r^2} \phi'^2 \right] + f'(\phi) \left[ 2\phi'^2 \dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi'^3 \right] \tag{20}$$

It is difficult to solve these equations explicitly, and so we assume  $P(r)$  is an arbitrary function of  $r$  and  $Q(t)$  is an arbitrary function of  $t$ , and write

$$\phi(r, t) = P(r)Q(t) \tag{21}$$

It may be noted that  $f(\phi)$  is an arbitrary function of  $\phi$ , so we can write,

$$f(\phi) = f_0 \phi^{-2} \tag{22}$$

where,  $f_0$  is a constant. This is a particular form of Galileon gravity rather than the most general form of Galileon gravity. As the general form of Galileon gravity was very complicated,

we simplified our analysis by assuming this particular form of Galileon gravity. It is possible to obtain analytic solutions in this particular form of Galileon gravity. We assume that the barotropic equation of state holds for the matter fluid

$$p = k\rho \quad (23)$$

where ' $k$ ' is a constant. The solution for  $Q(t)$  can be written as

$$Q(t) = \alpha_1 e^{-\lambda t} \quad (24)$$

Here  $\alpha_1$  and  $\lambda$  are arbitrary constants. It is not possible to obtain a similar solution for  $P(r)$  as the field equations are very complicated. So, we assume that

$$P(r) = \alpha r^n \quad (25)$$

where  $\alpha$  and  $n$  are arbitrary constants. We use these values of  $P$  and  $Q$  in the field equations and considering  $f_0 = 1$  (without much loss of generality in the given context). Thus, we obtain the following differential equation

$$\begin{aligned} r^2 m'' + \left[ 4k \frac{G(E)}{F(E)} + n(2+k) \right] r m' + [n \{ 2(k+1)(n-1) - (5k+6) \}] m + 2n[(3-n)(k+1)r \\ + (\omega + k + 2)\lambda r^2] - 3k \frac{G(E)}{F(E)} r = 0 \end{aligned} \quad (26)$$

Now we obtain an explicit expression for  $m$  by solving these differential equations,

$$m(t, r) = f_1(t)r^{\omega_1} + f_2(t)r^{\omega_2} + \frac{2n(n-3)(k+1) + 3k \frac{G(E)}{F(E)}}{(1-\omega_1)(1-\omega_2)} r - \frac{2n\lambda(\omega + k + 2)}{(2-\omega_1)(2-\omega_2)} r^2 \quad (27)$$

where

$$\begin{aligned} \omega_1, \omega_2 = & \left[ 1 - 4k \frac{G(E)}{F(E)} - n(2+k) \right] \\ & \pm \sqrt{\left\{ 4k \frac{G(E)}{F(E)} + n(2+k) - 1 \right\}^2 - 4n \{ 2(k+1)(n-1) - (5k+6) \}} \end{aligned} \quad (28)$$

Here  $f_1(t)$  and  $f_2(t)$  are arbitrary functions of  $t$ .

So, the deformed metric (7) can be expressed as

$$\begin{aligned} ds^2 = & \frac{1}{F(E)^2} [-1 + f_1(t)r^{\omega_1-1} + f_2(t)r^{\omega_2-1} \\ & + \frac{2n(n-3)(k+1) + 3k \frac{G(E)}{F(E)}}{(1-\omega_1)(1-\omega_2)} - \frac{2n\lambda(\omega + k + 2)}{(2-\omega_1)(2-\omega_2)} r] dt^2 \\ & + \frac{1}{F(E)G(E)} dt dr + \frac{1}{G(E)^2} r^2 d\Omega_2^2 \end{aligned} \quad (29)$$

which is the Rainbow deformed generalized Vaidya metric in Galileon gravity. It may be noted that this solution represents a special class of solution of the general model. This is because the general solution was very complicated, and so we made this assumption to simplify our analysis.

### 3 Collapse Study

In the previous section, we analyzed the Rainbow deformation of the Vaidya metric in Galileon gravity. In this section, we will analyze the gravitational collapse in this theory. We can let  $ds^2 = 0$  in Eq. (7), and obtain the equation for outgoing radial null geodesics. It may be noted that  $d\Omega_2^2 = 0$ , and

$$\frac{dt}{dr} = \frac{F(E)}{G(E) \left(1 - \frac{m(t,r)}{r}\right)}. \quad (30)$$

Thus, the central singularity exists at the point  $r = 0$ ,  $t = 0$ . Now we can study the behavior of the function  $X = \frac{t}{r}$  as it approaches this singularity at  $r = 0$ ,  $t = 0$  along the radial null geodesic. Let us denote this limiting value by  $X_0$ , and so we can write

$$\begin{aligned} X_0 &= \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} X = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{t}{r} = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{dt}{dr} = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{F(E)}{G(E) \left(1 - \frac{m(t,r)}{r}\right)} \end{aligned} \quad (31)$$

Now from Eqs. (27) and (31), we obtain

$$\begin{aligned} \frac{2}{X_0} &= \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{2G(E)}{F(E)} \left[ 1 - f_1(t)r^{\omega_1-1} - f_2(t)r^{\omega_2-1} \right. \\ &\quad \left. - \frac{2n(n-3)(k+1) + 3k \frac{G(E)}{F(E)}}{(1-\omega_1)(1-\omega_2)} + \frac{2n\lambda(\omega+k+2)}{(2-\omega_1)(2-\omega_2)} \frac{r}{t} \right] \end{aligned} \quad (32)$$

Here  $f_1(t) = \delta t^{-(\omega_1-1)}$  and  $f_2(t) = \epsilon t^{-(\omega_2-1)}$ , where  $\delta$  and  $\epsilon$  are constants. Now the equation for  $X_0$  can be written as

$$\delta X_0^{2-\omega_1} + \epsilon X_0^{2-\omega_2} - \left[ 1 - \frac{2n(n-3)(k+1) + 3k \frac{G(E)}{F(E)}}{(1-\omega_1)(1-\omega_2)} \right] X_0 + 2 \left[ 1 + \frac{n\lambda(\omega+k+2)}{(2-\omega_1)(2-\omega_2)} \right] = 0 \quad (33)$$

It may be noted that the outgoing null geodesic exists for  $X_0 > 0$ . Thus, a black hole will be formed when none of the solutions of this equation are positive. It is difficult to find analytic solutions for  $X_0$ , and so we will find numerical solutions for  $X_0$ . This will be done by assigning specific numerical values to the constants associated with this model. We will also need to use



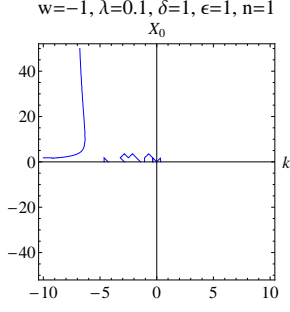


Fig.1

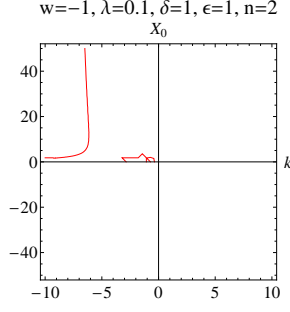


Fig.2

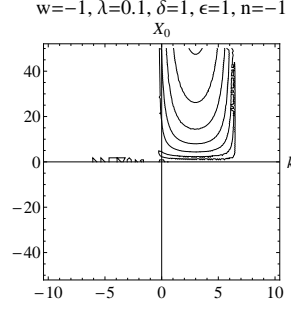


Fig.3

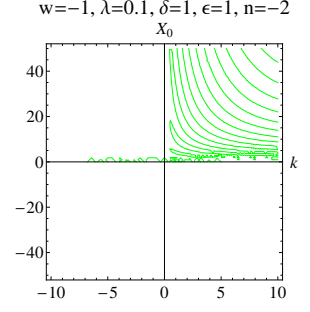


Fig.4

**Figs 1, 2, 3 and 4** show the variation of  $X_0$  with  $k$  for different values of  $n$  and for  $w = -1$

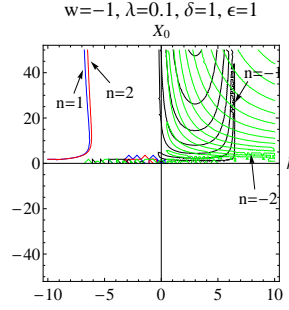


Fig.5

**Fig 5** shows the combined effect of figs.1,2,3 and 4.

a specific form of the rainbow function for performing this numerical analysis. Thus, we will use the rainbow functions motivated from loop quantum gravity approach and  $\kappa$ -Minkowski non-commutative spacetime [29, 30],

$$F(E/E_p) = 1, \quad G(E/E_p) = \sqrt{1 - \eta \left( \frac{E}{E_p} \right)} \quad (34)$$

In the above expressions,  $E_p$  is the Planck energy and it is given by  $E_p = 1/\sqrt{G} = 1.221 \times 10^{19}$  GeV. The behavior of the roots of this equation can be obtained from contour plots of  $X_0$  vs  $k$ , for fixed values of other parameters. Thus, we will be able to understand the behavior of the collapse at different cosmological eras. We can also understand the role played by other parameters in the collapse by adjusting the values of those parameters.

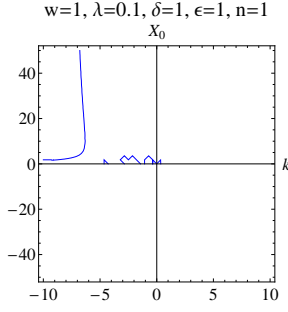


Fig.6

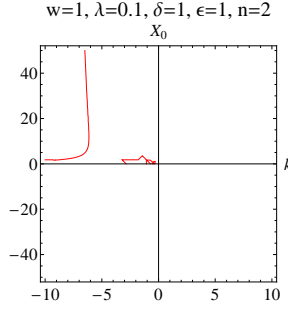


Fig.7

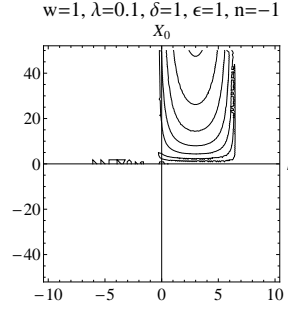


Fig.8

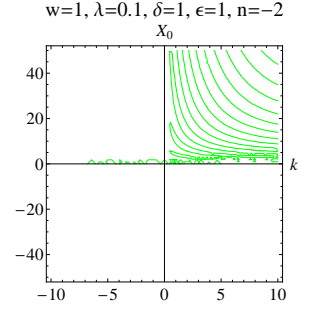


Fig.9

**Figs 6, 7, 8 and 9** show the variation of  $X_0$  with  $k$  for different values of  $n$  and for  $w = 1$

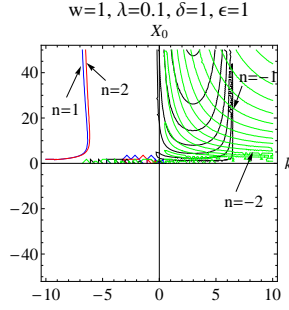


Fig.10

**Fig 10** shows the combined effect of figs.6,7,8 and 9.

## 4 Discussions and Conclusions

We have set the Galileon parameter  $w = -1$  in figs. (1), (2), (3) and (4), to obtain the contour plots of  $X_0$  vs  $k$ . This has been done by using different values of  $n$ , and fixed all other parameters such as  $\delta$ ,  $\lambda$ , and  $\epsilon$ . The positive solutions for  $X_0$  can be obtained in different cosmological eras for all these cases. Thus, in figs. (1) and (2), it was observed that for  $n = 1, 2$ , positive solutions exist when  $k < -1$ . So, for  $n = 1, 2$ , positive solution exists in a phantom DE era (late universe). However, it was also observed in figs. (3) and (4), that for  $n = -1, -2$ , positive solutions exist when  $k > 0$ . So, for  $n = -1, -2$ , positive solutions exist in a radiation era (early universe). We have also set the Galileon parameter  $w = 1$ , and obtained similar results in figs. (6), (7), (8) and (9). It was observed that even the range of  $X_0$  was similar for  $w = -1$  and  $w = 1$ . Thus, the collapsing system in rainbow deformed Galileon gravity does not depend on the Galileon parameter  $w$ . A similar result was obtained from the study of gravitational collapse in the usual Galileon gravity [14]. It can be observed that naked singularities are formed in the late universe, and black holes are formed in the early universe, for positive values of  $n$ . However, naked singularities are formed in the early universe, and black holes are formed in the late universe, for negative values of  $n$ . Similar

results are obtained from figs. (5) and (10) for different scenarios. In this paper, we first deformed Galileon gravity using rainbow functions. Then we analyzed the collapsing system in this Galileon gravity's rainbow. It was observed that the collapsing system does not depend on the Galileon parameter  $w$  in rainbow deformed Galileon gravity. It will be interesting to analyze other systems using a combination of gravity's rainbow with Galileon gravity.

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